



Patni, M., Minera Rebullá, S., Weaver, P., & Pirrera, A. (2017). *On the accuracy of the displacement-based Unified Formulation for modelling laminated composite beam structures*. Abstract from 20th International Conference on Composite Structures, Paris, France.

Peer reviewed version

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On the accuracy of the displacement-based Unified Formulation for modelling laminated composite beam structures

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Accurate modelling of composite structures is important to develop high performance components for many industrial applications. This approach provides better understanding of the behavior of composites under different loading conditions. The classical laminated beam theory, based on the Euler-Bernoulli hypothesis, is inaccurate for relatively thick laminated beams as three-dimensional (3D) effects such as transverse shear and normal deformations are neglected. To capture 3D stress fields for orthotropic and multi-layered composite structures accurately, the following modelling issues must be addressed:

1. Higher-order transverse shear deformations due to low orthotropy ratio, which further gives rise to the “stress-channelling” effect on axial stress [1].
2. Static inconsistencies at clamped ends, i.e. inaccurate stress prediction towards the clamped boundaries due to the presence of Kirchhoff rotations in the displacement field of higher-order theories [2].
3. Interlaminar continuity (IC) conditions on displacements and transverse stresses, i.e. displacements must be C^0 -continuous [3].
4. Zig-zag effect due to the transverse anisotropy, i.e. the difference in layerwise transverse shear and normal moduli, leads to sudden change in the slope of the three displacement fields at layer interfaces. This forces displacements to be C^1 -discontinuous.

High-fidelity 3D finite element models are often employed for accurate structural analysis. These models are computationally expensive when used for laminates with large number of layers, in optimization studies, or for non-linear analyses. To address these issues, a one-dimensional refined model for the analysis of laminated beam structures is presented. The flexural response of laminated composite beams is analyzed using the Unified Formulation [4] based on the newly developed Serendipity Lagrange elements. Like all, displacement-based weak form formulation with the principle of virtual displacements, Serendipity Lagrange elements do not obtain continuous transverse stresses. To tackle this issue and capture accurate 3D stress fields, a post-processing stress recovery step is proposed [5]. Our study highlights the advantages of using Serendipity

Lagrange expansions for laminated composite beams over finite elements, Taylor and Lagrange type models. Results show an excellent agreement with 3D finite element solutions obtained with ANSYS and are also compared with those given in the literature. For computationally inexpensive calculations, the present work also deals with a node-based variable expansion model: node dependent kinematics (NDK). The benefits of using this technique with the Unified Formulation are discussed. Serendipity Lagrange expansions (SLE) are shown to improve on commonly-used Taylor and Lagrange expansions, when model building is brought into consideration. Various numerical examples on beam-like structures are studied to demonstrate the effectiveness of NDK, when used in conjunction with SLE, in capturing the local effects at reduced computational cost.

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